

January 25, 2002

Electromagnetic Corrections to Charged Pion Scattering at Low Energies

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Abstract

The electromagnetic corrections to the low energy scattering amplitude involving charged pions only are investigated at leading and next-to-leading orders in the two-flavour chiral expansion. As an application, the corresponding variation in the strong $2S - 2P$ level shift is evaluated. The relative variation is of the order of 5%.

Keywords: Electromagnetic corrections, Pion Pion scattering, Chiral perturbation theory

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1. The study of hadronic atoms has become a very active field (see [1] for a recent account). Many experiments are devoted to measure the characteristics of such atoms with high precision [2, 3, 4, 5, 6]. These experimental results carry a well founded theoretical interest, since they provide a direct access to hadronic scattering lengths, leading, in this way, to valuable informations concerning the fundamental properties of QCD at low energy. For instance, the presently running DIRAC experiment aims at measuring the ponium lifetime τ with 10% accuracy [2]. This would allow one to determine the difference $a_0^0 - a_0^2$ with 5% precision by means of the Deser-type [7] relation [8]

$$\tau^{-1} \propto (a_0^0 - a_0^2)^2, \quad (1)$$

where a_l^I denotes the l -wave $\pi\pi$ scattering length in the channel with total isospin I . On the other hand, chiral perturbation theory (ChPT) predictions for the scattering lengths have reached a precision amounting to 2% [9]. Before confronting the experimental determination to the ChPT prediction, it is necessary to get all sources of corrections to the relation (1), valid in the absence of isospin breaking, under control. In this connection, bound state calculations were performed using different approaches, like potential scattering theory [10, 11], 3D-constraint field theory [12], Bethe-Salpeter equation [13] and non-relativistic effective lagrangians [14, 15]. For a review on the subject and a comparison between the various methods we refer the reader to [16]. Within the framework of non-relativistic effective lagrangians, the correct expression of relation (1) which include all isospin breaking effects at leading order (LO) and next-to-leading order (NLO) was given as [17]

$$\tau^{-1} = \frac{1}{9} \alpha^3 (4M_{\pi^\pm}^2 - 4M_{\pi^0}^2 - M_{\pi^\pm}^2 \alpha^2)^{\frac{1}{2}} \mathcal{A}^2 (1 + K). \quad (2)$$

In the preceding equation, $\alpha = e^2/(4\pi)$ stands for the fine-structure constant, \mathcal{A} and K possess the following expansions [17] in powers of the isospin breaking parameter $\kappa \in [\alpha, (m_d - m_u)^2]$

$$\mathcal{A} = -\frac{3}{32\pi} \text{Re } A_{\text{thr.}}^{+-;00} + o(\kappa), \quad (3)$$

$$K = \frac{1}{9} \left(\frac{M_{\pi^\pm}^2}{M_{\pi^0}^2} - 1 \right) (a_0^0 + 2a_0^2)^2 - \frac{2\alpha}{3} (\ln \alpha - 1) (2a_0^0 + a_0^2) + o(\kappa). \quad (4)$$

The quantity of interest,

$$-\frac{3}{32\pi} \text{Re } A_{\text{thr.}}^{+-;00} = a_0^0 - a_0^2 + h_1(m_d - m_u)^2 + h_2\alpha, \quad (5)$$

represents the real part of the $\pi^+\pi^- \rightarrow \pi^0\pi^0$ scattering amplitude at order κ , calculated at threshold within ChPT to any chiral order and from which we subtract the singular pieces behaving like q^{-1} and $\ln q$ with q being the center-of-mass three-momentum of the charged pions. The coefficient h_2 was calculated in [18] at next-to-leading order in the chiral expansion while $h_1 = \mathcal{O}(m_u + m_d)$ [16].

The DIRAC proposal [2] also mentioned the possibility to measure the strong $2S - 2P$ energy level shift ΔE_{strong} of the ponium. How this measurement could be performed in practice has been discussed in Ref. [19]. A simultaneous measurement of τ and ΔE_{strong} would allow to pin down a_0^0 and a_0^2 separately, since [20]

$$\Delta E_{\text{strong}} \propto 2a_0^0 + a_0^2. \quad (6)$$

Bound state calculation of the isospin breaking corrections to (6) were done in [11] using potential scattering theory (the main contribution to the total level shift comes from vacuum polarization effects, see e.g. [21]). Non-relativistic effective lagrangian calculations concerning ΔE_{strong} are not available for pionium, but exist for the pionic hydrogen case [22]. One may thus reasonably expect an expression similar to Eq. (2)

$$\begin{aligned}\Delta E_{\text{strong}} &= -\frac{1}{8} \alpha^3 M_{\pi^\pm} \mathcal{A}'(1 + K'), \\ \mathcal{A}' &= \frac{1}{32\pi} \text{Re } A_{\text{thr.}}^{+-;+-} + o(\kappa),\end{aligned}\tag{7}$$

involving the $\pi^+\pi^- \rightarrow \pi^+\pi^-$ scattering amplitude $A^{+-;+-}$. Electromagnetic corrections to this scattering amplitude will be calculated at next-to-leading order in the present work.

2. The elastic scattering process

$$\pi^+(p_+) + \pi^-(p_-) \rightarrow \pi^+(p'_+) + \pi^-(p'_-),\tag{8}$$

is studied in terms of the Lorentz invariant Mandelstam variables

$$s = (p_+ + p_-)^2, \quad t = (p_+ - p'_+)^2, \quad u = (p_+ - p'_-)^2,$$

satisfying the on-shell relation $s + t + u = 4M_{\pi^\pm}^2$. These variables are related to the center-of-mass three-momentum q and scattering angle θ by

$$s = 4(M_{\pi^\pm}^2 + q^2), \quad t = -2q^2(1 - \cos\theta), \quad u = -2q^2(1 + \cos\theta).\tag{9}$$

Let $A^{+-;+-}$ and $A^{++;++}$ denote the respective scattering amplitudes for the process (8) and for the crossed channel reaction $\pi^+\pi^+ \rightarrow \pi^+\pi^+$. Then, $s \leftrightarrow u$ crossing is expressed as

$$A^{+-;+-}(s, t, u) = A^{++;++}(u, t, s).\tag{10}$$

We shall calculate the scattering amplitude (10) at NLO including electromagnetic effects. The strong sector chiral lagrangian for two-flavour ChPT was constructed in [23]. Treating isospin violation of electromagnetic origin requires the extension of ChPT in order to include virtual photons. This can be done by building operators in which photons figure as explicit dynamical degrees of freedom. The electromagnetic sector of the chiral lagrangian for two-flavour ChPT has been discussed at NLO in [18] and [24]. We shall work in the $m_u = m_d$ limit and use the lagrangian representation of [18]. At one-loop accuracy, all of the following chiral orders are present: $p^2, e^2, p^4, e^2p^2, e^4$. We do however not consider the $\mathcal{O}(e^4)$ contributions, which include two-photon exchange box diagrams, and which are expected to be smaller than the other contributions at the same order. Using Feynman graph techniques, the amplitude (10) can be represented at NLO by the diagrams³ depicted in Fig. 1.

The scattering amplitude (10) is conveniently written in the following $s \leftrightarrow t$ symmetric decomposition

$$\begin{aligned}A^{+-;+-}(s, t, u) &= \left\{ \frac{s - M_{\pi^0}^2}{F^2} + B^{+-;+-}(s, t, u) + C^{+-;+-}(s, t, u) \right. \\ &\quad \left. + e^2 \left(\frac{u - t}{s} \right) [F_V^\pi(s)]^2 \right\} + \{s \leftrightarrow t\}.\end{aligned}\tag{11}$$

³If one uses the so-called σ -model parametrization of the pion fields, the diagrams (e) of Fig. 1 and (b) of Fig. 2 vanish identically.

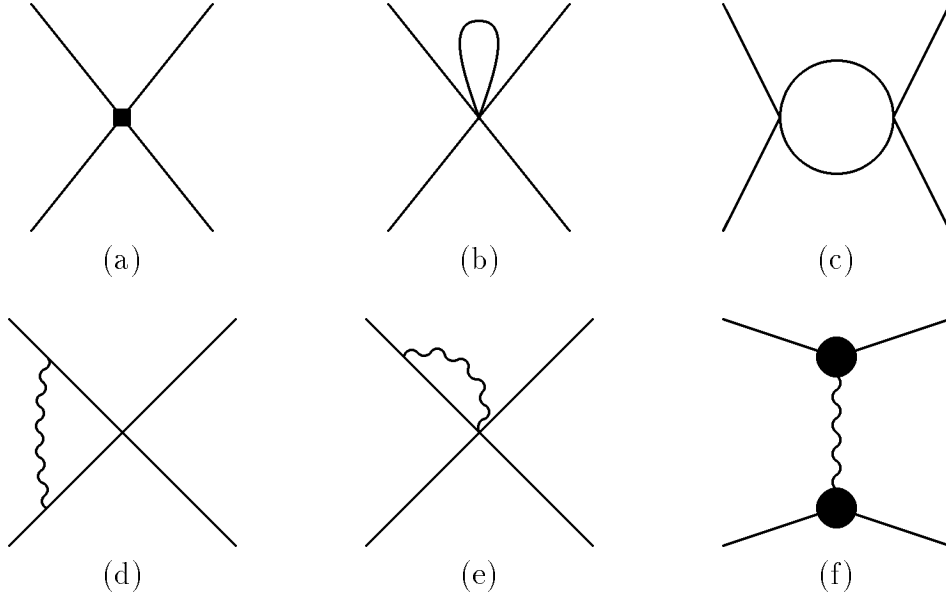


Figure 1: The various topologies of Feynman diagrams contributing to the charged $\pi\pi$ scattering amplitude at order one loop, but ignoring $\mathcal{O}(e^4)$ effects. The full circles appearing in the Born-type diagram (f) are made explicit in Fig. 2.

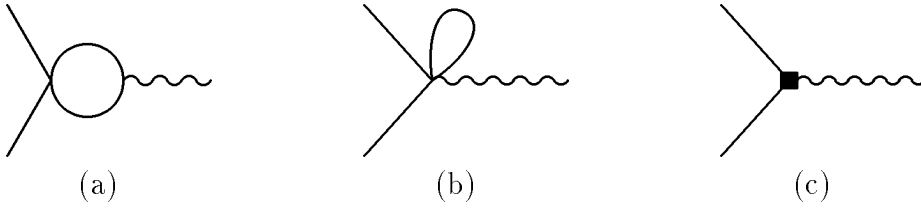


Figure 2: The electromagnetic vertex function of a charged pion to one-loop order. The full square takes into account the contribution from the low-energy constants just as the tree contribution including the effect of wave function renormalization. Diagrams of order $\mathcal{O}(e^3 p)$ are discarded.

In the preceding expression, $B^{+-;+-}$ collects the unitarity pieces arising from the diagrams of type (c) and (d) in Fig. 1,

$$\begin{aligned}
B^{+-;+-}(s, t, u) &= \frac{1}{2F^4} (s - M_{\pi^0}^2)^2 \bar{J}_{00}(s) \\
&+ \frac{1}{F^4} \left[\frac{s^2}{4} - \frac{1}{12} (u - t)(s - 4M_{\pi^\pm}^2) + 2s\Delta_\pi + 4\Delta_\pi^2 \right] \bar{J}_{+-}(s) \\
&+ \frac{1}{4F^4} (u - 2M_{\pi^\pm}^2 - 2\Delta_\pi)(u - 2M_{\pi^\pm}^2 - 2\Delta_\pi - 4e^2 F^2) \bar{J}_{+-}(u) \\
&+ \frac{2e^2}{F^2} (u - 2M_{\pi^\pm}^2 - 2\Delta_\pi) [2(s - 2M_{\pi^\pm}^2)G_{+-\gamma}(s) - (u - 2M_{\pi^\pm}^2)G_{+-\gamma}(u)] \\
&- \frac{e^2}{F^2} \left[s + 4\Delta_\pi - 4(s - 2M_{\pi^\pm}^2) \left(\frac{t - u}{t + u} \right) \right] \bar{J}_{+-}(s), \tag{12}
\end{aligned}$$

where

$$\Delta_\pi = M_{\pi^\pm}^2 - M_{\pi^0}^2.$$

The expressions for the various loop functions appearing in (12) can be found in [18]. The function $C^{+-;+-}$ represents the contributions from tadpoles as well as from the strong [23] and electromagnetic [18] low-energy constants

$$\begin{aligned}
C^{+-;+-}(s, t, u) &= \frac{s - M_{\pi^0}^2}{F^2} \frac{e^2}{32\pi^2} \left[-18 - 8 \left(1 + \ln \frac{m_\gamma^2}{M_{\pi^\pm}^2} \right) + \frac{1}{2} (\mathcal{K}^{+-;+-} - \mathcal{K}^{++;++}) \right] \\
&+ \frac{e^2 M_{\pi^0}^2}{32\pi^2 F^2} \left[10 + \frac{1}{2} (\mathcal{K}^{+-;+-} + \mathcal{K}^{++;++}) \right] - \frac{e^2}{2\pi^2 F^2} (s - 2M_{\pi^\pm}^2) \left(\frac{t - u}{t + u} \right) \\
&+ \frac{1}{48\pi^2 F^4} [(s - 2M_{\pi^\pm}^2)^2 (\bar{l}_1 + \bar{l}_2) + (u - 2M_{\pi^\pm}^2)^2 \bar{l}_2] - \frac{M_{\pi^0}^4}{32\pi^2 F^4} \bar{l}_3 \\
&+ \frac{1}{16\pi^2 F^4} \left(-\frac{5}{18} u^2 - \frac{13}{18} s^2 + \frac{2}{3} u M_{\pi^0}^2 + \frac{19}{6} u \Delta_\pi + \frac{5}{18} M_{\pi^0}^4 - \frac{58}{9} M_{\pi^0}^2 \Delta_\pi \right) \\
&- \frac{1}{96\pi^2 F^4} \frac{\Delta_\pi}{M_{\pi^0}^2} (-3s^2 + 16sM_{\pi^0}^2 + 2uM_{\pi^0}^2 - 23M_{\pi^0}^4). \tag{13}
\end{aligned}$$

In this last expression, we have dropped all contributions of order $\mathcal{O}(e^4)$ and beyond. In particular, we have expanded all logarithms of the pion mass ratio,

$$\ln \frac{M_{\pi^\pm}^2}{M_{\pi^0}^2} = \frac{\Delta_\pi}{M_{\pi^0}^2} + \dots$$

The terms $\mathcal{K}^{+-;+-}$ and $\mathcal{K}^{++;++}$ involve combinations of the electromagnetic counterterms k_i ,

$$\mathcal{K}^{+-;+-} = \left(3 + \frac{4Z}{9} \right) \bar{k}_1 - \frac{40Z}{9} \bar{k}_2 - 9\bar{k}_3 + 4Z\bar{k}_4 + 4(1 + 8Z)\bar{k}_6 + 2(1 - 8Z)\bar{k}_8, \tag{14}$$

$$\mathcal{K}^{++;++} = - \left(3 + \frac{4Z}{9} \right) \bar{k}_1 - \frac{248Z}{9} \bar{k}_2 + 9\bar{k}_3 - 20Z\bar{k}_4 + 4(1 + 8Z)\bar{k}_6 + 2(1 - 8Z)\bar{k}_8. \tag{15}$$

The low-energy constants \bar{l}_i and \bar{k}_i are related to the respective renormalized running couplings $l_i^r(\mu) \equiv l_i^r$ and $k_i^r(\mu) \equiv k_i^r$ at the scale $\mu = M_{\pi^\pm}$,

$$l_i^r = \frac{\gamma_i}{32\pi^2} \left(\bar{l}_i + \ln \frac{M_{\pi^\pm}^2}{\mu^2} \right), \quad k_i^r = \frac{\sigma_i}{32\pi^2} \left(\bar{k}_i + \ln \frac{M_{\pi^\pm}^2}{\mu^2} \right),$$

where γ_i and σ_i are the renormalization group beta-functions whose values are given in [23] and [18], respectively. The last quantity that remains to be defined in (11) is F_V^π , which denotes the electromagnetic pion form factor

$$\langle \pi^+(p'_+)|\mathcal{J}^\mu(0)|\pi^+(p_+)\rangle = -ie(p_+ + p'_+)^\mu F_V^\pi(Q^2), \quad Q = p'_+ - p_+,$$

with \mathcal{J}^μ being the electromagnetic current. At the order we are working, it is given by [23, 25]

$$F_V^\pi(p^2) = 1 + \frac{p^2}{6F^2} \left[\left(1 - \frac{4M_{\pi^\pm}^2}{p^2} \right) \bar{J}_{+-}(p^2) + \frac{1}{16\pi^2} \left(\bar{l}_6 - \frac{1}{3} \right) \right]. \quad (16)$$

Finally, let us mention that the scattering amplitude (11) is scale independent but infrared divergent. This infrared divergence is signaled by the presence of a $\ln(m_\gamma^2/M_{\pi^\pm}^2)$ term, where m_γ is a small photon mass acting as an infrared regulator. These infrared divergencies have to cancel in observable quantities, like the total cross section with soft photon emission. These divergencies do however not show up in the discussion of the electromagnetic corrections to the strong level shift, to which we now turn.

3. The scattering lengths a_0^0 and a_0^2 are well-defined quantities in the absence of radiative corrections. Their NLO expressions were derived in [23] and, for historical reasons [26], we shall reproduce them in terms of the charged pion mass

$$\begin{aligned} a_0^0 &= \frac{7M_{\pi^\pm}^2}{32\pi F_\pi^2} \left\{ 1 + \frac{5M_{\pi^\pm}^2}{84\pi^2 F_\pi^2} \left[\bar{l}_1 + 2\bar{l}_2 - \frac{3}{8}\bar{l}_3 + \frac{21}{10}\bar{l}_4 + \frac{21}{8} \right] \right\}, \\ a_0^2 &= -\frac{M_{\pi^\pm}^2}{16\pi F_\pi^2} \left\{ 1 - \frac{M_{\pi^\pm}^2}{12\pi^2 F_\pi^2} \left[\bar{l}_1 + 2\bar{l}_2 - \frac{3}{8}\bar{l}_3 - \frac{3}{2}\bar{l}_4 + \frac{3}{8} \right] \right\}, \end{aligned} \quad (17)$$

with F_π being the pion decay constant whose NLO expression is given in the isospin limit by [23]

$$F_\pi = F \left(1 + \frac{M_{\pi^0}^2}{16\pi^2 F^2} \bar{l}_4 \right).$$

We are interested in electromagnetic corrections to the *strong* energy level shift. To this end, one has to consider the part of the amplitude corresponding to one-photon-irreducible (OPI) diagrams. It is then convenient to subtract the contribution from diagram (f) of Fig. 1 as follows

$$\begin{aligned} A^{+\mp;+\mp}(s, t, u) &= A_{\text{OPI}}^{+\mp;+\mp}(s, t, u) + A_{\text{Born}}^{+\mp;+\mp}(s, t, u), \\ A_{\text{Born}}^{+-;+-}(s, t, u) &= e^2 \left\{ \frac{u-t}{s} [F_V^\pi(s)]^2 + \frac{u-s}{t} [F_V^\pi(t)]^2 \right\}, \\ A_{\text{Born}}^{++;++}(s, t, u) &= e^2 \left\{ \frac{s-t}{u} [F_V^\pi(u)]^2 + \frac{s-u}{t} [F_V^\pi(t)]^2 \right\}. \end{aligned} \quad (18)$$

Next, we expand the real part of $A_{\text{OPI}}^{+\mp;+\mp}$ in powers of q ,

$$\text{Re } A_{\text{OPI}}^{+-;+-}(s, t, u) = \frac{M_{\pi^0}^2}{F_\pi^2} \frac{e^2}{4} \frac{M_{\pi^0}}{q} + \text{Re } A_{\text{thr.}}^{+-;+-} + \mathcal{O}(q), \quad (19)$$

$$\text{Re } A_{\text{OPI}}^{++;++}(s, t, u) = \frac{M_{\pi^0}^2}{F_\pi^2} \frac{e^2}{4} \frac{M_{\pi^0}}{q} + \text{Re } A_{\text{thr.}}^{++;++} + \mathcal{O}(q). \quad (20)$$

The term in q^{-1} is due to the Coulomb photon exchanged between the scattered particles in diagram (d) of Fig. 1. It should be absorbed in the static characteristics of pionium [8, 27, 28] within the treatment of the bound state properties. Furthermore, the infrared divergent terms show up only in the $\mathcal{O}(q)$ terms. The regular terms, $\text{Re } A_{\text{thr.}}^{+-;+-}$ and $\text{Re } A_{\text{thr.}}^{++;++}$, constitute the central object of the current work. Their isospin limits are nothing else than $2a_0^0 + a_0^2$ and a_0^2 respectively,

$$\frac{1}{32\pi} \text{Re } A_{\text{thr.}}^{+-;+-} = \frac{1}{6} (2a_0^0 + a_0^2) + \Delta a_0(+--;+-), \quad (21)$$

$$\frac{1}{32\pi} \text{Re } A_{\text{thr.}}^{++;++} = a_0^2 + \Delta a_0(++; ++). \quad (22)$$

The NLO expressions for the electromagnetic corrections are found to be

$$32\pi \Delta a_0(+--;+-) = \frac{2\Delta_\pi}{F_\pi^2} + \frac{M_{\pi^0}^2 \Delta_\pi}{8\pi^2 F_\pi^4} (2 + \bar{l}_3) - \frac{e^2 M_{\pi^0}^2}{16\pi^2 F_\pi^2} (24 - \mathcal{K}^{+-;+-}), \quad (23)$$

$$32\pi \Delta a_0(++;++) = \frac{2\Delta_\pi}{F_\pi^2} + \frac{M_{\pi^0}^2 \Delta_\pi}{16\pi^2 F_\pi^4} (3 + 2\bar{l}_3 + 8\bar{l}_4) - \frac{e^2 M_{\pi^0}^2}{16\pi^2 F_\pi^2} (20 - \mathcal{K}^{++;++}), \quad (24)$$

with the combinations of the electromagnetic low-energy constants defined in (14) and (15).

4. For the numerical estimates of Eqs. (23) and (24), we use the following values [29]. The charged and neutral pions masses are $M_{\pi^\pm} = 139.570 \text{ MeV}$ and $M_{\pi^0} = 134.976 \text{ MeV}$, respectively. The pion decay constant is taken as $F_\pi = 92.4 \text{ MeV}$. The values for the strong sector low-energy constants are: $\bar{l}_3 = 2.9 \pm 2.4$ [23], $\bar{l}_4 = 4.4 \pm 0.3$ [30]. As for the two-flavour low-energy constants \bar{k}_i , no direct numerical estimates exist up to now. On the other hand, some of the corresponding three-flavour counterterms $K_i^r(\mu) \equiv K_i^r$ were evaluated in [31, 32, 33]. In order to give a numerical estimate of the combinations (14) and (15), it is then necessary to relate the k_i^r 's to the K_i^r 's (note that the combinations of k_i^r 's involved in the amplitude $\pi^+\pi^- \rightarrow \pi^0\pi^0$ were expressed in terms of the K_i^r 's in [12] and [15, 16]). In the present case, this requires to compute the corresponding three-flavour representations of the quantities $\text{Re } A_{\text{thr.}}^{+-;+-}$ and $\text{Re } A_{\text{thr.}}^{++;++}$ [34], and then to match them with Eqs. (21) and (22), respectively, in the limit of a very massive strange quark. With the help of the strong sector matching relations [35] between the l_i^r 's and the L_i^r 's, we obtain

$$\begin{aligned} \mathcal{K}^{+-;+-} &= \frac{64\pi^2}{9} [-12(K_1^r + K_2^r) + 9(6K_3^r + K_4^r) - 10(K_5^r + K_6^r) + 72(K_8^r + K_{10}^r + K_{11}^r)] \\ &- 6Z_0 \left(1 + \ln \frac{B_0 m_s}{\mu^2} \right) - 16Z_0 \left(\bar{l}_4 + 2 \ln \frac{M_{\pi^\pm}^2}{\mu^2} \right), \end{aligned} \quad (25)$$

$$\begin{aligned} \mathcal{K}^{++;++} &= \frac{64\pi^2}{9} [12(K_1^r - 5K_2^r) - 9(6K_3^r + 5K_4^r) + 2(5K_5^r - 31K_6^r) + 72(K_8^r + K_{10}^r + K_{11}^r)] \\ &- 2Z_0 \left(1 + \ln \frac{B_0 m_s}{\mu^2} \right) - 16Z_0 \left(\bar{l}_4 - \ln \frac{M_{\pi^\pm}^2}{\mu^2} \right) - 12 \ln \frac{M_{\pi^\pm}^2}{\mu^2}, \end{aligned} \quad (26)$$

where Z_0 is the three-flavour analogue of Z ,

$$Z = \frac{\Delta_\pi}{2e^2 F^2} \Big|_{m_u=m_d=0}, \quad Z_0 = \frac{\Delta_\pi}{2e^2 F_0^2} \Big|_{m_u=m_d=m_s=0}.$$

Using the three-flavour to two-flavour matching relations [35]

$$\begin{aligned} F &= F_0 \left(1 - \frac{\bar{M}_K^2}{32\pi^2 F_0^2} \ln \frac{\bar{M}_K^2}{\mu^2} + \frac{8\bar{M}_K^2}{F_0^2} L_4^r \right), \\ B &= B_0 \left[1 - \frac{\bar{M}_\eta^2}{96\pi^2 F_0^2} \ln \frac{\bar{M}_\eta^2}{\mu^2} + \frac{16\bar{M}_K^2}{F_0^2} (2L_6^r - L_4^r) \right], \end{aligned}$$

with

$$\bar{M}_K^2 = B_0 m_s, \quad \bar{M}_\eta^2 = \frac{4}{3} B_0 m_s, \quad B_0 = -\frac{\langle \bar{u}u \rangle}{F_0^2} \Big|_{m_u=m_d=m_s=0},$$

together with the three-flavour [36] and two-flavour [18] calculations of Δ_π we get

$$Z = Z_0 \left(1 - \frac{32\bar{M}_K^2}{F_0^2} L_4^r \right) + \frac{4\bar{M}_K^2}{F_0^2} K_8^r.$$

Note that the following replacements are valid at the order we are working

$$Z = Z_0 \rightarrow \frac{\Delta_\pi}{2e^2 F_\pi^2}, \quad B_0 m_s \rightarrow M_{K^\pm}^2 - \frac{1}{2} M_{\pi^\pm}^2,$$

with $M_{K^\pm} = 493.677$ MeV. Using the values of the K_i^r 's derived in [31] at $\mu = M_\rho = 770$ MeV and assigning to each of them an uncertainty of $\pm 1/(16\pi^2)$ coming from naïve dimensional analysis, we obtain for the combinations (25) and (26)

$$\frac{e^2 M_{\pi^0}^2}{F_\pi^2} \mathcal{K}^{+-;+-} = 8.63 \pm 12.02, \quad \frac{e^2 M_{\pi^0}^2}{F_\pi^2} \mathcal{K}^{++;++} = -15.13 \pm 14.62. \quad (27)$$

This entails (we first show separately the contributions of each of the three terms in Eqs. (23) and (24))

$$\begin{aligned} \Delta a_0(+--;+-) &= 2.9 \cdot 10^{-3} + (1.9 \pm 0.9) \cdot 10^{-4} + (2.5 \pm 7.6) \cdot 10^{-4} \\ &= (3.2 \pm 0.8) \cdot 10^{-3}, \\ \Delta a_0(++;++) &= 2.9 \cdot 10^{-3} + (8.7 \pm 1.1) \cdot 10^{-4} + (-12.0 \pm 9.2) \cdot 10^{-4} \\ &= (2.5 \pm 0.9) \cdot 10^{-3}. \end{aligned} \quad (28)$$

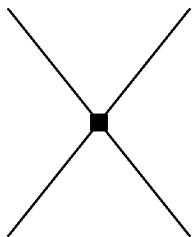
From the last result, we see that the size of electromagnetic corrections to the combination $2a_0^0 + a_0^2$ represents $\sim 5\%$ of the NLO value. This amounts to one-half of the size of the next-to-next-to-leading order (NNLO) strong interaction corrections [9]. Also, the bulk of the corrections arises from the electromagnetic pion mass difference, whereas the uncertainties are dominated by the error bars on the determinations of $\mathcal{K}^{+-;+-}$ and $\mathcal{K}^{++;++}$.

5. In the present work, the scattering amplitude for the process $\pi^+\pi^- \rightarrow \pi^+\pi^-$ was calculated at LO and NLO in the chiral counting and in the presence of electromagnetic corrections. We have also evaluated the influence of the latter on the corresponding combination $2a_0^0 + a_0^2$ of S -wave $\pi\pi$ scattering lengths which is relevant for the $2S-2P$ strong energy level shift of ponium. The LO electromagnetic corrections come entirely from the difference between the charged and neutral pions masses. As for the NLO electromagnetic corrections, their size amounts to $\sim 10\%$ of the one for the lowest order isospin breaking effects. This corresponds to a 5% correction to the strong $2S-2P$ level shift at NLO.

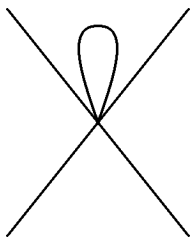
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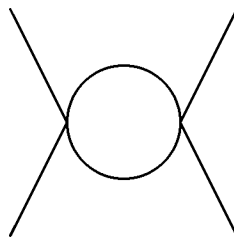
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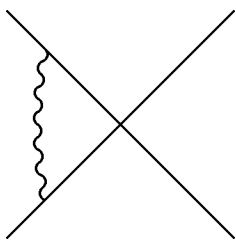
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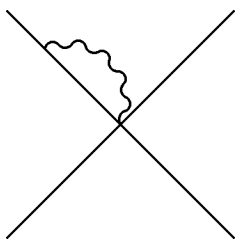
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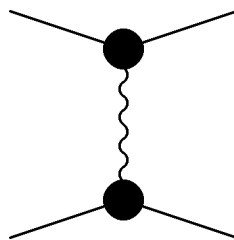
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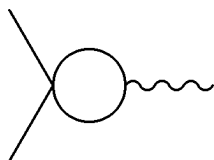
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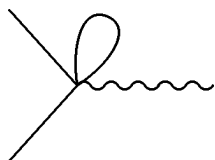
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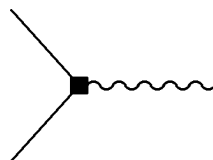
(f)



(a)



(b)



(c)